PEME5409

POWER PLANT ENGINEERING

Module 1: Section 3

Flow Through Nozzles

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Nozzles

- A nozzle is a passage of smoothly varying cross-section by means of which the pressure Energy of working fluid is converted into kinetic energy.

- The shape of the nozzle is designed such that it will perform this conversion of energy with minimum loss.

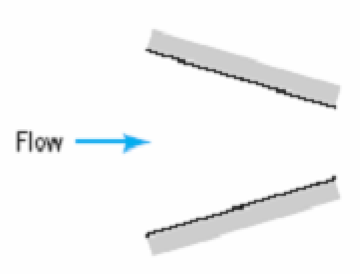
Applications

- Steam, water and gas turbines.
- Jet engine s, rocket motors to produce thrust.
- Artificial fountains.
- Flow measurements.
- Injectors for pumping feed water to the boilers and ejectors for removing air from the condensers.

When a fluid is decelerated in a passage of varying cross section causing rise in pressure along the stream, then the passage is called a diffuser. These are extensively used in centrifugal compressors, axial flow compressors, ram jets etc.

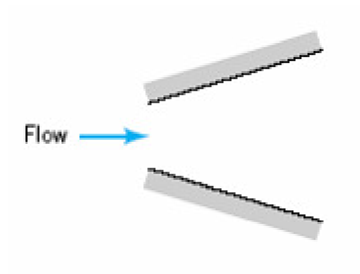
Types of Nozzles

i) Convergent Nozzle



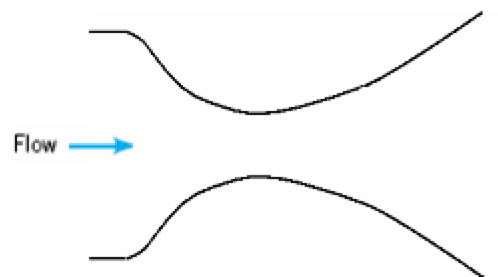
Cross section of the nozzle decreases continuously
From entrance to exit

ii) Divergent Nozzle



Cross section of the nozzle increases continuously
From entrance to exit

iii) Convergent –Divergent Nozzle



Cross section of the nozzle first decreases and then increases

Specific volume of a liquid is almost constant over a wide range of pressure, therefore, Nozzles for liquids are always convergent, even at high exit velocities (e.g. a fire-hose)

Relationship between Area, Velocity and Pressure in Nozzle flow

Applying mass conservation equation

$$\rho AV = Constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$d(\rho AV) = 0$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Applying momentum conservation equation

$$\frac{dp}{\rho} + VdV + gdz = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$dp = -\rho V dV$$

$$\frac{dp}{\rho V^2} = -\frac{dV}{V} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Relationship between Area, Velocity and Pressure in Nozzle flow

Substituting (4) in (2) and simplifying

$$\frac{dA}{A} = \frac{dp}{\rho V^2} - \frac{d\rho}{\rho}$$

$$\frac{dA}{A} = \frac{dp}{\rho V^2}\left(1 - \frac{V^2}{dp/d\rho}\right) \text{....................}(5)$$

Invoking the relation ($c^2 = dp/d\rho$) where *c* is sound velocity

$$\frac{dA}{A} = \frac{dp}{\rho V^2}\left(1 - \frac{V^2}{c^2}\right) = \frac{dp}{\rho V^2}\left(1 - M^2\right) \text{..}(6)$$

we see that for M<1 an area change causes a pressure change of the same sign, i.e. positive d*A* means positive d*p* for M<1. For M>1, an area change causes a pressure change of opposite sign.

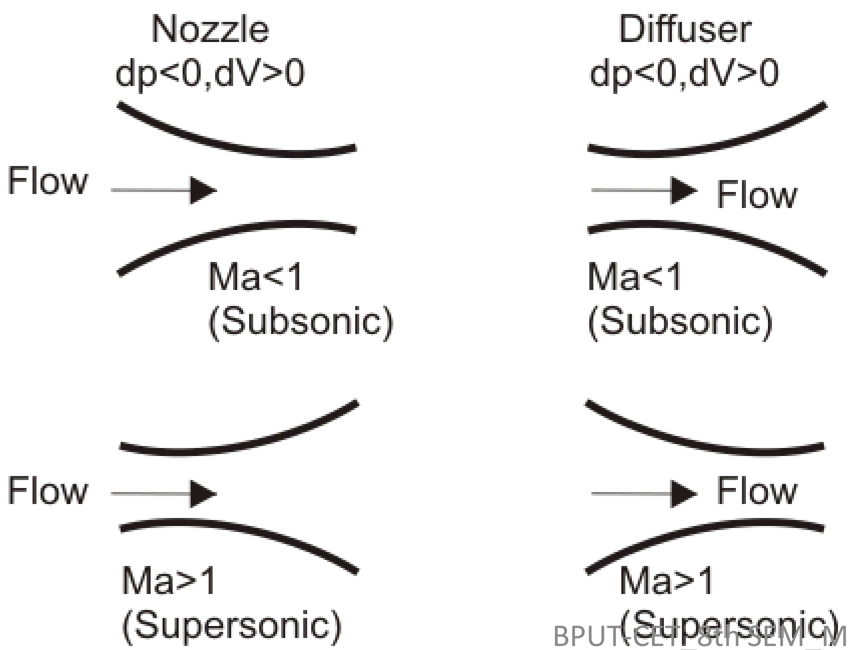
$$\frac{dA}{A} = -\frac{dV}{V}\left(1 - M^2\right) \text{..........................}(7)$$

we see that M<1 an area change causes a velocity change of opposite sign, i.e. positive d*A* means negative d*V* for M<1. For M>1, an area change causes a velocity change of same sign.

Relationship between Area, Velocity and Pressure in Nozzle flow

•At subsonic speeds (M<1) ,a decrease in area increases the speed of flow. A subsonic nozzle should have a convergent profile and a subsonic diffuser should possess a divergent profile. The flow behaviour in the regime of M<1 is therefore qualitatively the same as in incompressible flows.

•In supersonic flows (M>1), the effect of area changes are different.
A supersonic nozzle must be built with an increasing area in the flow direction. A supersonic diffuser must be a converging channel.
Divergent nozzles are used to produce supersonic flow in missiles and launch vehicles.

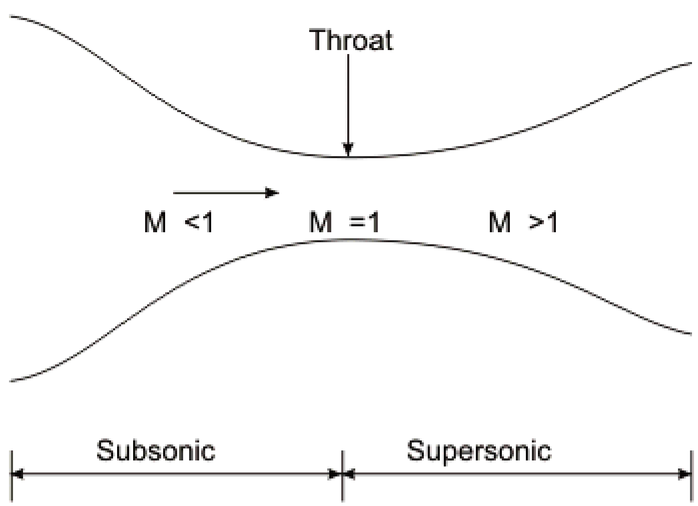


Shapes of nozzles and diffusers in subsonic and supersonic regimes

Relationship between Area, Velocity and Pressure in Nozzle flow

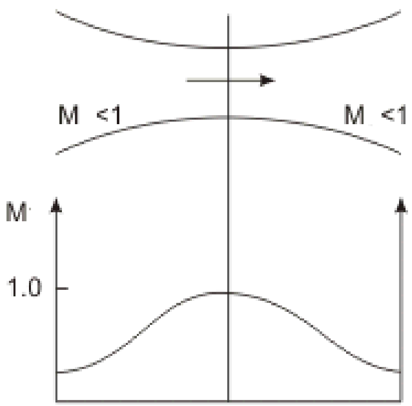
Suppose a nozzle is used to obtain a supersonic stream staring from low speeds at the inlet
.
Then the Mach number should increase from M=0 near the inlet to M>1 at the exit.
It is clear that the nozzle must converge in the subsonic portion and diverge in the supersonic portion. Such a nozzle is called a *convergent-divergent nozzle or de Laval nozzle*

From Figure below, it is clear that the Mach number must be unity at the throat, where the area is neither increasing nor decreasing. This is consistent with Eq. 7 which shows that d*V* can be non-zero at the throat only if M=1. It also follows that the sonic velocity can be achieved only at the throat of a nozzle or a diffuser.



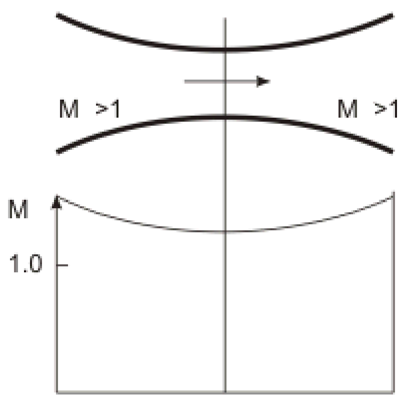
A convergent-divergent nozzle

Relationship between Area, Velocity and Pressure in Nozzle flow



The flow in a convergent-divergent duct may be subsonic everywhere with *M* increasing in the convergent portion and decreasing in the divergent portion with at the throat The first part of the duct is acting as a nozzle, whereas the second part is acting as a diffuser.

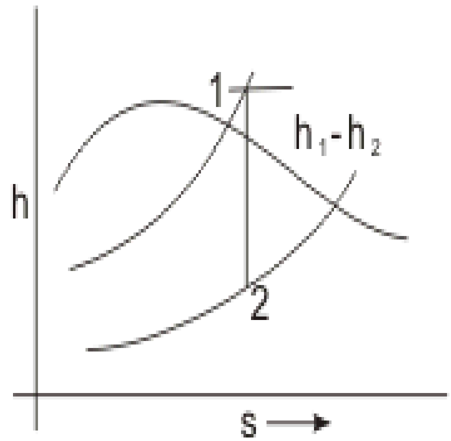
A convergent-divergent nozzle with M≠1

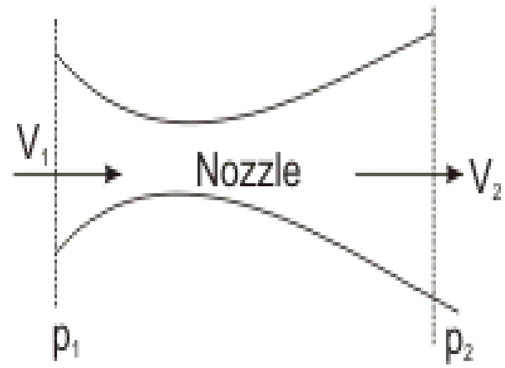


we may have a convergent-divergent duct in which the flow is supersonic everywhere with *M* decreasing in the convergent part and increasing in the divergent part and again at the throat

A convergent-divergent nozzle with M≠1

Isentropic flow of a vapor/gas through a nozzle





Using SFEE between inlet and exit of the nozzle

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$V_2 = \sqrt{2(h_1 - h_2)}$$

Isentropic flow of a vapor/gas through a nozzle

We know Tds equation,

$$Tds = dh - vdp$$

For the Isentropic flow,

$$dh = vdp$$

Integrating above equation

$$h_1 - h_2 = -\int_1^2 vdp$$

Assuming that the pressure and volume of steam during expansion obey the law pv^n = constant, where n is the isentropic index

$$-\int_1^2 vdp = -\int_1^2 \left(p_1 v_1^n\right)^{1/n} p^{-1/n} dp = -\int_1^2 \left(p_2 v_2^n\right)^{1/n} p^{-1/n} dp$$

Isentropic flow of a vapor/gas through a nozzle

$$= -\left\{ p_2^{\frac{1}{n}} v_2 \left[\frac{p^{\left(1-\frac{1}{n}\right)}}{1-\frac{1}{n}} \right]_1^2 \right\} = -\frac{n}{n-1} \left\{ p_2^{\frac{1}{n}} v^2 \left(p_2^{\frac{n-1}{n}} - p_1^{\frac{n-1}{n}} \right) \right\}$$

$$= -\frac{n}{n-1} \left\{ p_2 v_2 - p_1^{1/2} v_1 p_1^{\frac{n-1}{n}} \right\}$$

$$h_1 - h_2 = \frac{n}{n-1} \{ p_1 v_1 - p_2 v_2 \}$$

$$V_2 = \sqrt{\frac{2n}{n-1} \{ p_1 v_1 - p_2 v_2 \}} = \sqrt{\frac{2n}{n-1} p_1 v_1 \left\{ 1 - \frac{p_2 v_2}{p_1 v_1} \right\}}$$

$$= \sqrt{\frac{2n}{n-1} p_1 v_1 \left\{ 1 - \frac{p_2}{p_1} \left(\frac{p_1}{p_2} \right)^{1/n} \right\}} = \sqrt{\frac{2n}{n-1} p_1 v_1 \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right\}}$$

Isentropic flow of a vapor/gas through a nozzle

Using Mass conservation equation,

$$\dot{m} = \rho_2 A_2 V_2$$

$$\frac{\dot{m}}{A_2} = \rho_2 V_2 = \frac{V_2}{v_2}$$

$$\frac{\dot{m}}{A_2} = \frac{1}{v_2}\sqrt{\frac{2n}{n-1} p_1 v_1 \left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}}$$

$$= \sqrt{\frac{2n}{n-1} \frac{p_1}{v_1} \left\{\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right\}}$$

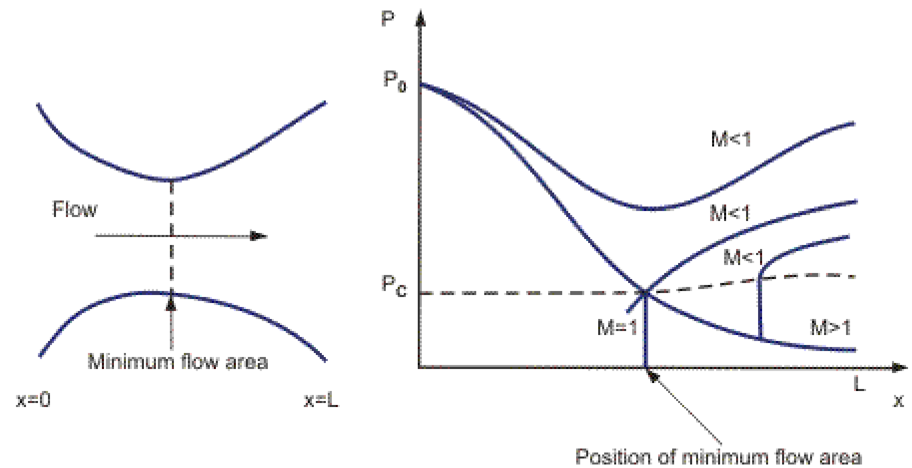
Isentropic flow of a vapor/gas through a nozzle

The exit pressure, p_2 determines the for a given inlet condition. The mass flow rate is maximum when,

$$\frac{d}{dy}\left[y^{\frac{2}{n}} - y^{\frac{n+1}{n}}\right] = 0; y = \frac{p_2}{p_1}$$

$$y = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}} or$$

$$\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$$



$\frac{p_2}{p_1}$ Is called as critical pressure ratio

$$\frac{p_2}{p_1} = \left[\frac{2}{n+1}\right]^{\frac{n}{n-1}}$$

n =1.3 for super saturated steam ; $\frac{p_2}{p_1}$=0.546

=1.135 for dry saturated steam ; $\frac{p_2}{p_1}$ =0.58

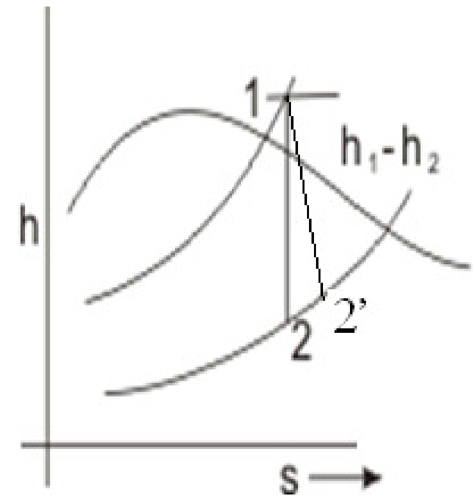
= 1.035+0.1x for wet steam with dryness fraction x

Entropy changes with friction

Considering a small change of state, the change in entropy

$$\delta s = \frac{\delta q_f}{T} = \frac{\delta w_f}{T}$$

Entropy $\uparrow$ Dryness fraction $\uparrow$



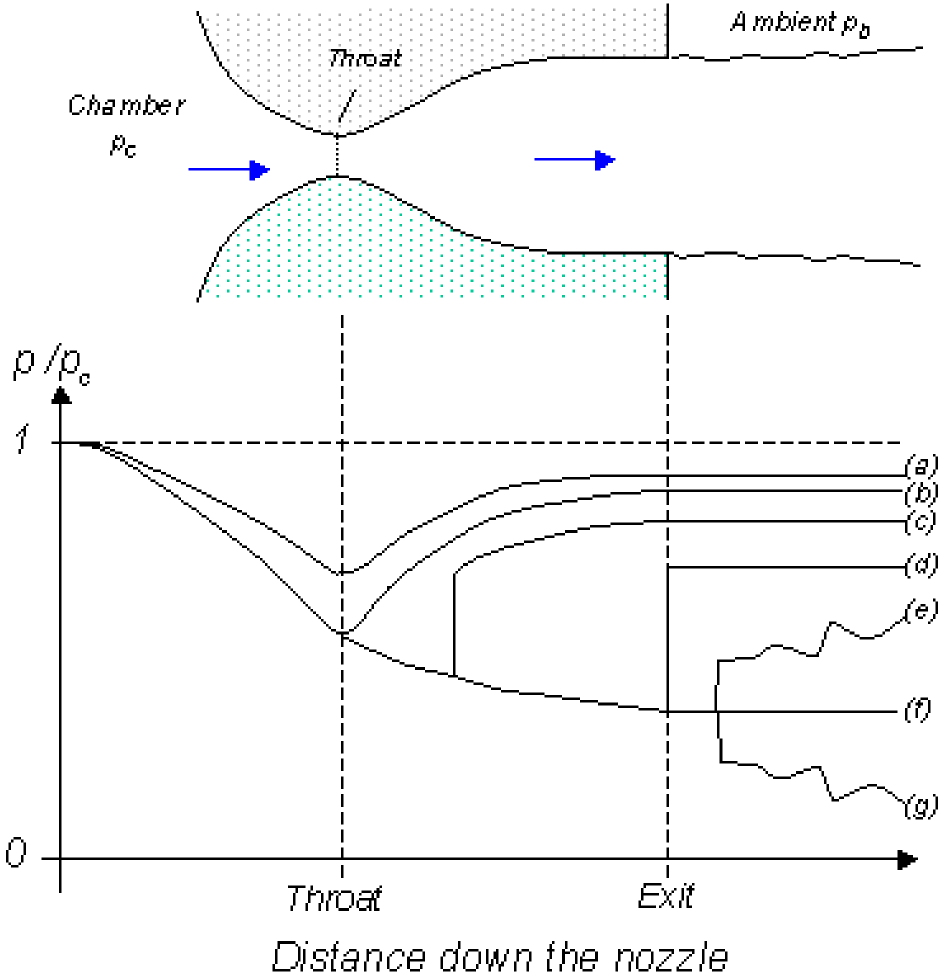
$$\eta_{nozzle} = \frac{h_1 - h_2^{'}}{h_1 - h_2}$$

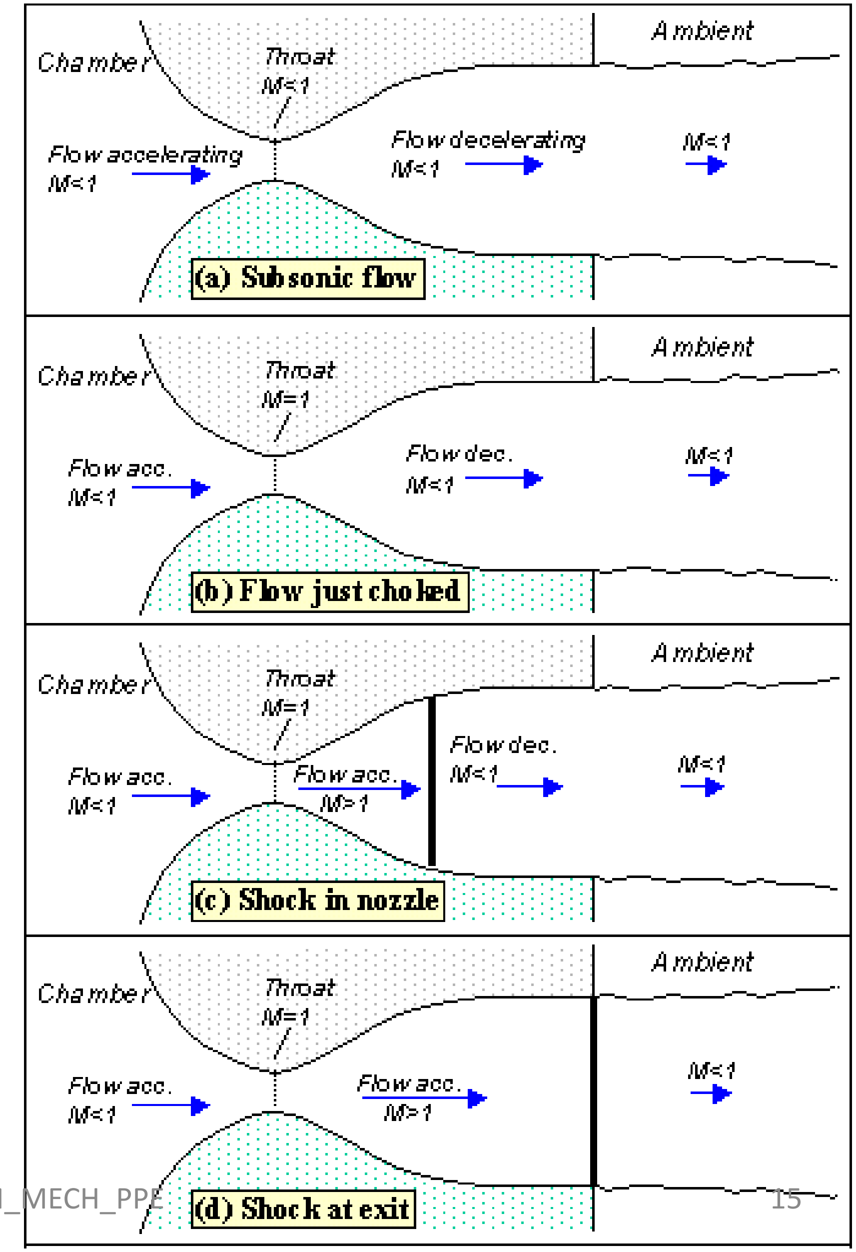
$$V_2 = \sqrt{2\eta_{nozzle}\left(h_1 - h_2\right)}$$

Reason of losses in nozzle

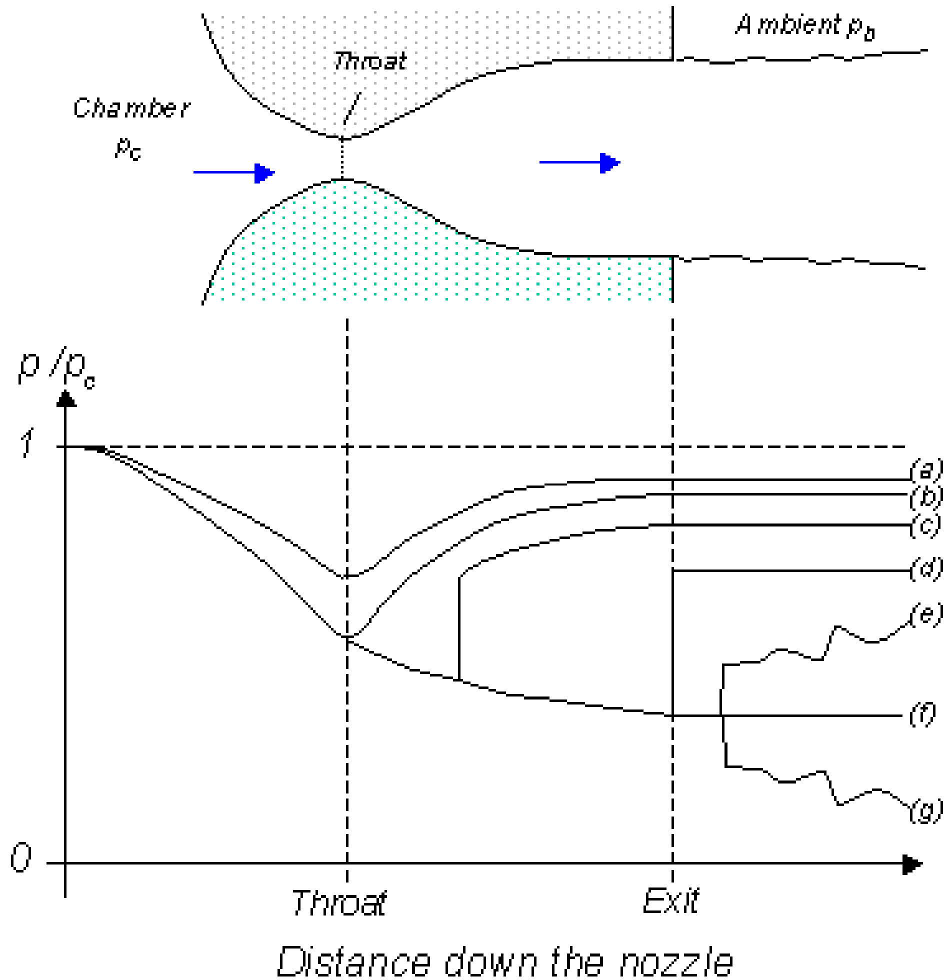
- the friction between the nozzle surface and steam
- the internal friction of steam itself
- The shock losses

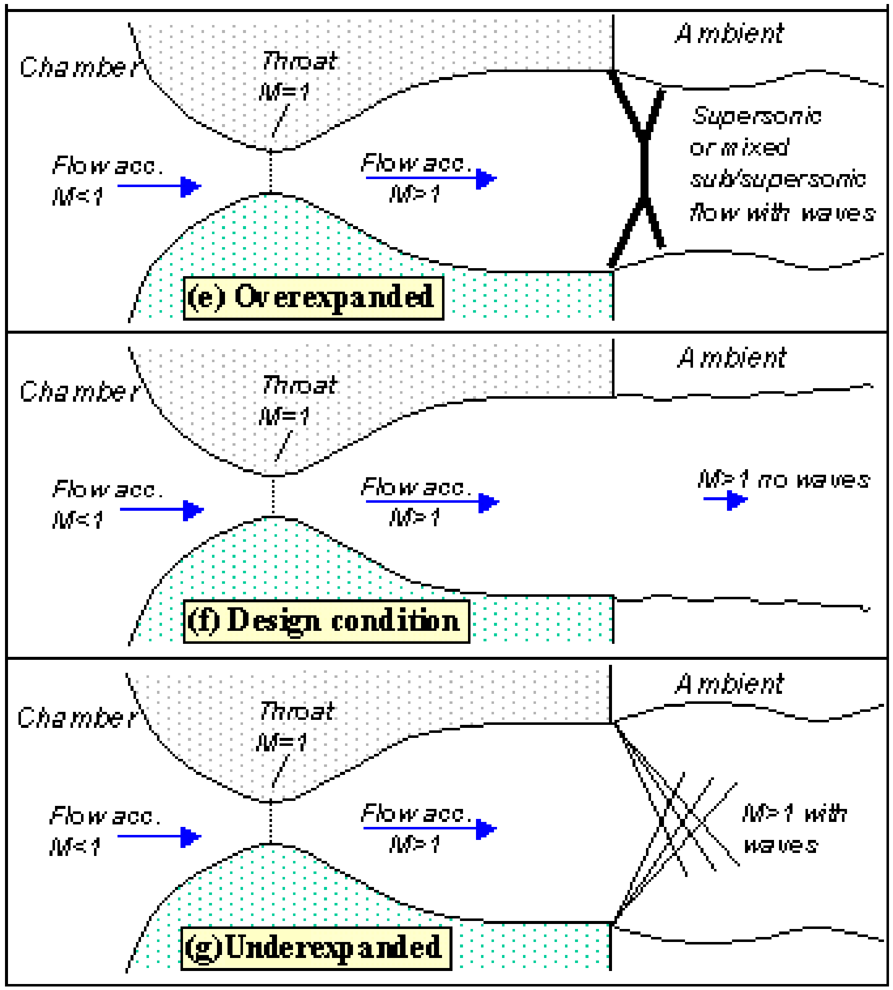
Operation of C-D Nozzle





Isentropic flow of a vapor/gas through a nozzle





All practical aerospace nozzles operate in regimes (e)-(g)

Operation of C-D Nozzle

- Figure (a) shows the flow through the nozzle when it is completely subsonic (i.e. nozzle isn't choked). The flow accelerates out of the chamber through the converging section, reaching its maximum (subsonic) speed at the throat. The flow then decelerates through the diverging section and exhausts into the ambient as a subsonic jet. Lowering the back pressure in this state increases the flow speed everywhere in the nozzle.

- Further lowering p_b results in figure (b). The flow pattern is exactly the same as in subsonic flow, except that the flow speed at the throat has just reached Mach 1. Flow through the nozzle is now choked since further reductions in the back pressure can't move the point of M=1 away from the throat. However, the flow pattern in the diverging section does change as the back pressure is lowered further.

- As p_b is lowered below that needed to just choke the flow a region of supersonic flow forms just downstream of the throat. Unlike a subsonic flow, the supersonic flow accelerates as the area gets bigger. This region of supersonic acceleration is terminated by a normal shock wave. The shock wave produces a near-instantaneous deceleration of the flow to subsonic speed. This subsonic flow then decelerates through the remainder of the diverging section and exhausts as a subsonic jet. In this regime if the back pressure is lowered or raised the length of supersonic flow in the diverging section before the shock wave increases or decreases, respectively.

Operation of C-D Nozzle

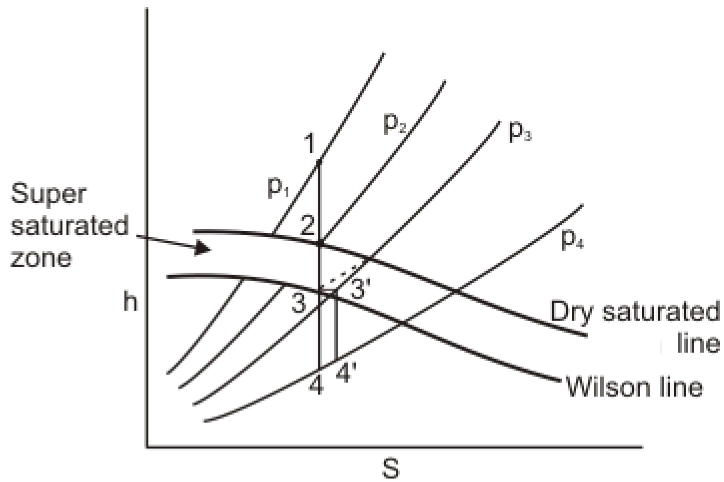
- If p_b is lowered enough the supersonic region may be extended all the way down the nozzle until the shock is sitting at the nozzle exit, figure (d). Because of the very long region of acceleration (the entire nozzle length) the flow speed just before the shock will be very large. However, after the shock the flow in the jet will still be subsonic.
- Lowering the back pressure further causes the shock to bend out into the jet, figure (e), and a complex pattern of shocks and reflections is set up in the jet which will now involve a mixture of subsonic and supersonic flow, or (if the back pressure is low enough) just supersonic flow. Because the shock is no longer perpendicular to the flow near the nozzle walls, it deflects it inward as it leaves the exit producing an initially contracting jet. We refer to this as over-expanded flow because in this case the pressure at the nozzle exit is lower than that in the ambient (the back pressure)- i.e. the flow has been expanded by the nozzle too much.
- A further lowering of the back pressure changes and weakens the wave pattern in the jet. Eventually, the back pressure will be lowered enough so that it is now equal to the pressure at the nozzle exit. In this case, the waves in the jet disappear altogether, figure (f), and the jet will be uniformly supersonic. This situation, since it is often desirable, is referred to as the 'design condition‘, Pe=Pa.

Operation of C-D Nozzle

- Finally, if the back pressure is lowered even further we will create a new imbalance between the exit and back pressures (exit pressure greater than back pressure), figure (g). In this situation, called under-expanded, expansion waves that produce gradual turning and acceleration in the jet form at the nozzle exit, initially turning the flow at the jet edges outward in a plume and setting up a different type of complex wave pattern.

- Summary Points to Remember:
  - When the flow accelerates (sub or supersonically) the pressure drops
  - The pressure rises instantaneously across a shock
  - The pressure throughout the jet is always the same as the ambient (i.e. the back pressure) unless the jet is supersonic and there are shocks or expansion waves in the jet to produce pressure differences
  - The pressure falls across an expansion wave

Supersaturated flow



Degree of super heat = $\frac{p_3}{p_{3s}}$

p_3 = limiting saturation pressure

p_{3s} = saturation pressure at temperature T_3 shown on T-s diagram

Degree of undercooling= T_{3s} - T_3

T_{3s} is the saturation temperature at

T_3 = Supersaturated steam temperature at point 3 which is the limit of supersaturation.

The increase in measure discharge to the theoretically calculated discharge is due to the time lag in the condensation of steam and thus the steam remains in dry instead of wet

This is called supersaturation.

Supersaturated flow

- There is an increase in entropy and specific volume of steam

- The heat drop is reduced below that for thermal equilibrium as a consequence the exit velocity of the steam is reduced.

- since the condensation does not take place during supersaturated expansion, so the temperature at which the supersaturation occurs will be less than the saturation temperature corresponding to the pressure.

- Density of supersaturated flow is more than that for the equilibrium conditions which gives the increase in the mass of steam discharged.

- The dryness fraction of steam is improved

References

Books recommended

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Web

www.wikipedia.com
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